MATHEMATICS SPECIALIST Year 12
Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section
Reading time for this section: 2 minutes
Working time for this section: 30 minutes
Marks available: 28 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1.   The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer Booklet using a blue/black pen. Do not use erasable/gel pens.

3.   Answer all questions.

4.   You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5.   Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6.   **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7.   It is recommended that **you do not use pencil**, except in diagrams.

Question 1 **(6 marks)**

If $y^4 = x^4 - 4xy - 7$, then determine

(a) $\frac{dy}{dx}$. (3 marks)

(b) the value of gradient at the point (2,1). (1 mark)

(c) the equation of the normal at the point (2,1). (2 marks)

See next page

Question 2   **(8 marks)**

(a)   Differentiate $y = (\sin x)^x$, using logarithmic differentiation, giving your answer in
      terms of x only.   (3 marks)

(b)   If $\frac{dA}{dt} = 6e^{2t}$ and $A = 4$ when $t = 0$, then

   (i)   determine A in terms of t.   (2 marks)

   (ii)   determine the exact value of A when $t = 0.5$.   (1 mark)

   (iii) use $\frac{dA}{dt}$ to determine the approximate change in A when t changes from 0 to
   0.01.   (2 marks)

Question 3   **(4 marks)**

Determine the exact value(s) of t, on the curve $x = t(t - 4)$, $y = t^2(t - 4)$ at which the gradient is $\frac{1}{2}$.

Question 4   **(10 marks)**

A raw milk tank truck has a cylindrical shaped tank that is 1 m in diameter and 8 m in length. The tank is being filled from empty to half-full at a rate of 0.1 m³ of raw milk per minute. The diagram below shows a cross section of the tank, where x is the depth of the raw milk level.
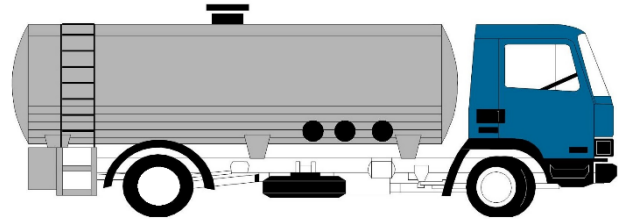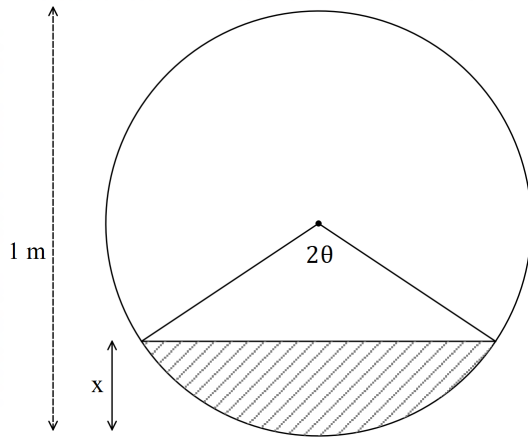

(a)   Show that the volume V $[m^3]$ of raw milk in the tank is given by $V = 2\theta - sin(2\theta)$, where $0 \leq \theta \leq \frac{\pi}{2}$.   (2 marks)

(b)   Using the equation in part (a), determine an expression for $\frac{dV}{d\theta}$ in terms of θ.   (1 mark)

Question 4 continued

(c) The diagram below shows a way to link the rising depth of the raw milk x with the angle θ and the radius of the circle. Use this diagram to express x in terms of θ, and hence show that: (3 marks)

$$\frac{dx}{dt} = \frac{1}{2} \sin \theta \, \frac{d\theta}{dt}$$
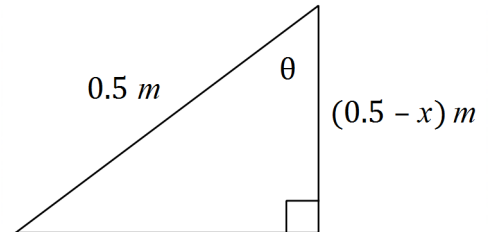
(d) Determine the exact rate at which the raw milk level is rising at the instant when the raw milk is 25 cm deep.   (4 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

MATHEMATICS SPECIALIST Year 12
Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section
Reading time for this section: 2 minutes
Working time for this section: 45 minutes
Marks available: 45 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in this assessment

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet using a blue/black pen. Do not use erasable/gel pens

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.
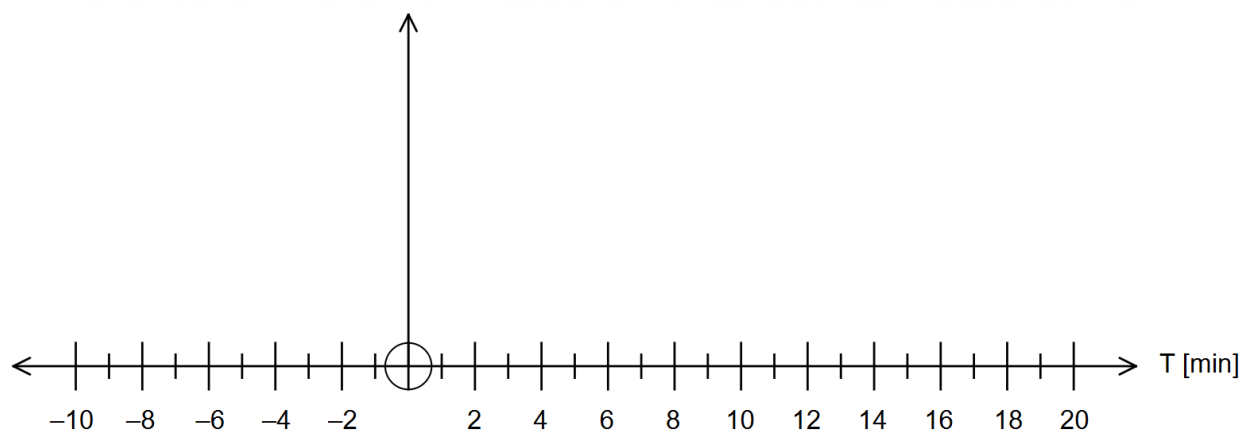
Question 5   **(6 marks)**

The "Red CAT" bus runs to an approximate schedule of 10 minutes between busses in the city centre. The time T in minutes that models the time interval between two buses (i.e. the time it takes for one bus to arrive after another one has departed) is modelled by a uniform distribution where $0 \leq T \leq 20$.
The population mean is $\mu(T) = 10$ minutes and the population variance is $\sigma^2(T) = 300$.
A sample of 30 bus intervals is taken and the sample mean $\bar{T}$ is calculated.

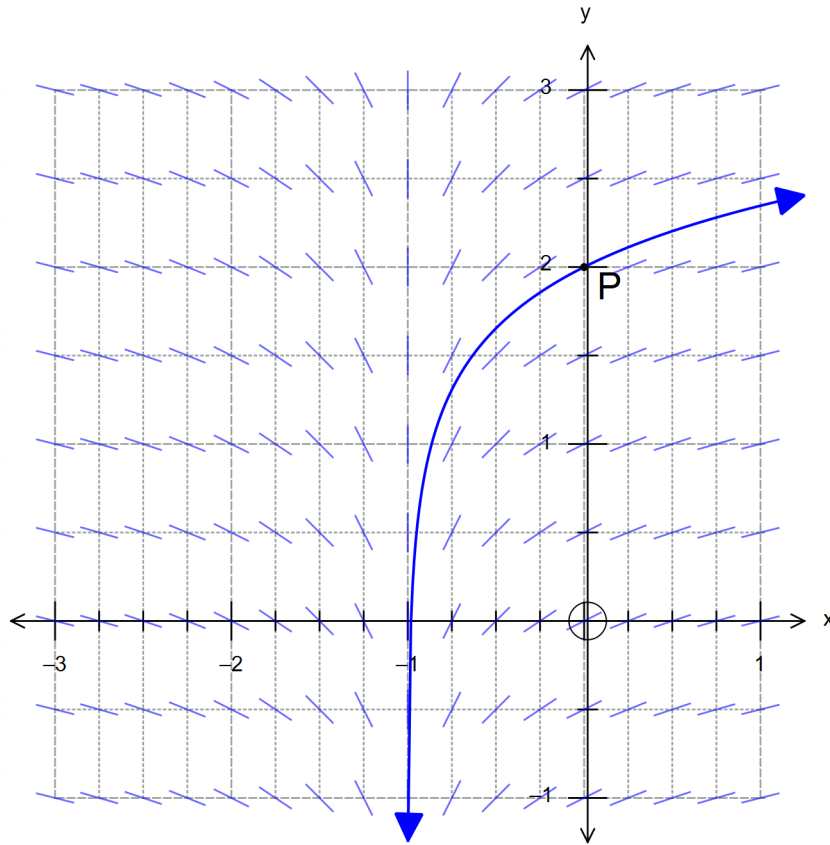(a)   Determine $P(5 \leq \bar{T} \leq 15)$ correct to **two (2)** decimal places.   (3 marks)

(b)   A large number of samples are taken, each with 30 bus interval measurements. Sketch the likely distribution of the sample mean $\bar{T}$. Indicate in your diagram the calculation from part (a).   (3 marks)

Question 6   **(6 marks)**

The slope field shown below is given by $\dfrac{dy}{dx} = \dfrac{1}{2x+2}$ with $x \neq -1$.

(a)   Determine the value of the slope field at the point $P(0, 2)$ shown.   (1 mark)

(b)   Explain the behaviour of the slope field along the line $x + 1 = 0$.   (1 mark)

See next page

Question 6 continued

(c)   Determine the equation of the solution shown that passes through $P(0, 2)$.

(4 marks)

Question 7   **(8 marks)**

(a) The displacement, x metres, of an object from an origin, O, is given by
$x = A\cos(kt)$, where A and k are constants. Prove that the object is moving with
simple harmonic motion and that it is initially at an extreme position.   (3 marks)

Question 7 continued

(b) The depth of water in a harbour, above and below the mean depth, is an example of simple harmonic motion. In a harbour, the low tide depth of 3 metres is recorded at 7 a.m. one morning and the next high tide is expected to record a depth of 15 metres at 1:20 p.m. later that same day. A container ship requires a depth of at least 5 metres for safe entry into the harbour, for unloading at the dockside and for leaving. Determine, to the nearest 5 minutes, the times between 7 a.m. and 9 p.m. that day, for which it is safe for the ship to engage in these activities.

(5 marks)

Question 8   **(7 marks)**

The diagram represents a container full of fluid that is punctured on the side. The fluid above the puncture line escapes due to pressure and gravity while the fluid below the line remains in the container.

The rate at which the fluid escapes is proportional to the amount of fluid left in the container, which can be modelled by the differential equation:

$$\frac{dQ}{dt} = k(100 - Q)$$
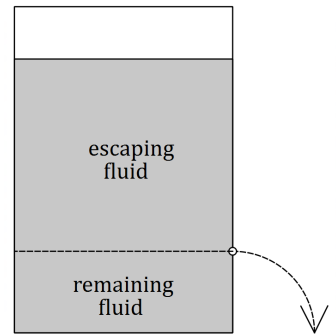
where Q represents the amount of fluid left in the container, in litres, and t is the time, in minutes, since the fluid began to leak.

(a)   Use calculus to show that the solution to this differential equation is given by
       $Q(t) = Ae^{-kt} + 100$, where A is a constant.   (3 marks)

Question 8 continued

The container originally holds 900 litres of fluid and it takes 2.5 hours for half of the fluid to escape.

(b)   Determine the total time, correct to the nearest minute, that it takes for the leak to stop. (4 marks)

Question 9   **(10 marks)**

The lifetime of antibiotics in the bloodstream of a patient is believed to be distributed as a logarithmic random variable, with mean $\mu = 8$ hours and a standard deviation of $\sigma = 8$ hours.

A random sample of the bloodwork of 40 patients is selected to analyse the speed at which the human body metabolises a particular type of antibiotic that is being trialled. The variable $\bar{X}$ represents the mean lifetime of the antibiotic in the bloodstream for these 40 patients.

(a)   State the distribution of the sample mean lifetime $\bar{X}$. Justify your answer.

                                                                     (3 marks)

(b)   Determine the probability that the sample mean lifetime is between 5 and 11 hours.   **(1 mark)**

Question 9 continued

Kathryn, the chief doctor in charge of the study, suggests that the lifetimes may not be logarithmically distributed, and that an alternative distribution would still have a mean of $\mu = 8$ hours and a standard deviation of $\sigma = 8$ hours.

(c)   If Kathryn is correct, will your answer to (b) change? Explain your answer.

(2 marks)

A different random sample of size n of the bloodwork of patients was selected. Repeated sampling of samples of size n shows that there is a 5% chance of obtaining a sample mean greater than 10 hours.

(d)   Determine the value of n.   (4 marks)

See next page

Question 10   **(8 marks)**

A scientific experiment measures the total bacteria present per hour over a few days.
The graph below represents the amount of bacteria $B(t)$ present for t hours.
Two particular measurements are provided for $t = 15$ hours and $t = 25$ hours.
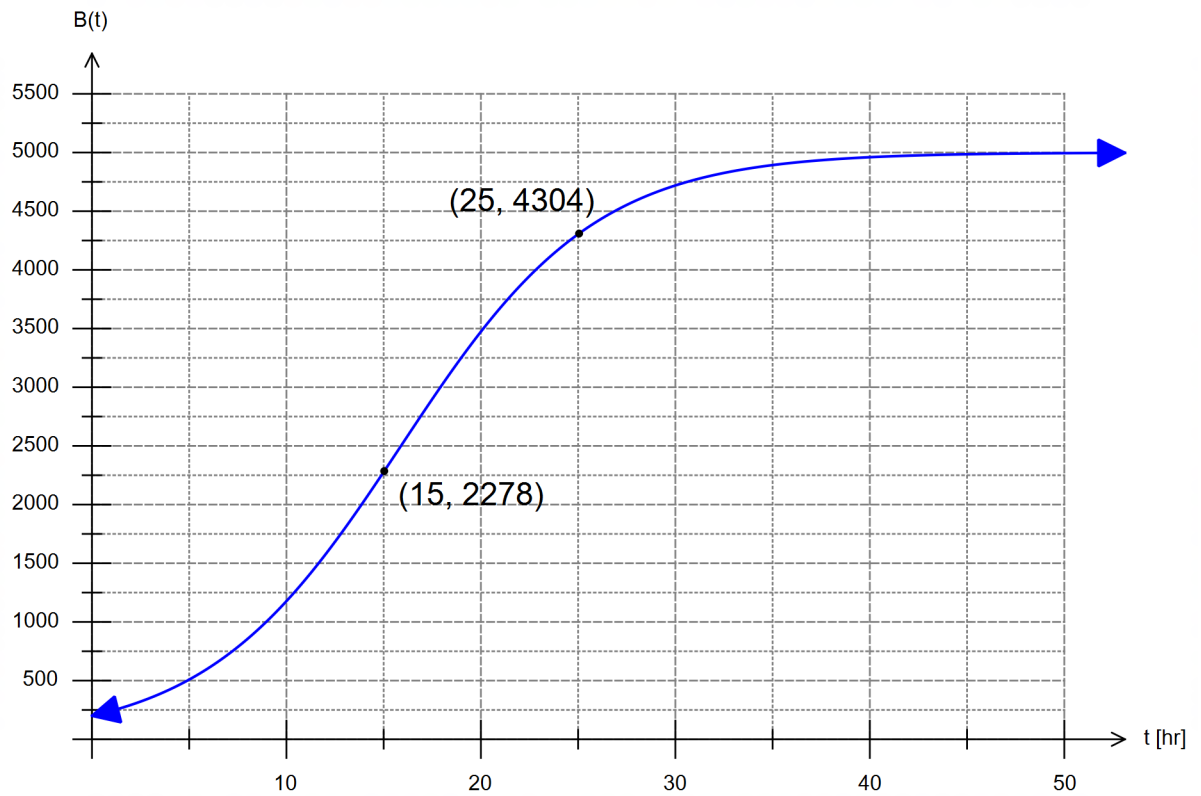
(a) Suggest a differential equation that would model $B(t)$, the number of bacteria present per hour.   (1 mark)

See next page

Question 10 continued

(b)   Determine the solution $B(t)$ to the differential equation in (a).   (4 marks)

(c)   How long would it take for the bacteria to reach its maximum capacity?   (3 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____